MAGIC FROM A SACRED CHINESE TURTLE, TOGETHER WITH POTENT MAGIC FROM INDIA, AND FROM THE HAND OF BENJAMIN FRANKLIN

Magical Arrays, With Varied Algorithms for Their Construction

Presented to

The Athenaeum Society

Hopkinsville, Kentucky

By

George M. Draper

December 3, 1992

When the world was much younger, in the times before radio, and television, before the Chinese and Gutenberg printing presses and widespread literacy, in the times even before myths crystallized into sacred beliefs and set men at incessant holy wars with each other, there was, even then, the need for man to amuse himself in some innocent and satisfying way.

There was always sex, high on the priority list, but sex of necessity was a sporadic diversion, not suitable for one's constant attention. Furthermore, it did not require any particular mental application and, with the awakening of man's inquiring intellect, his newfound ability to count, to communicate, and to assign abstract values, there was the burning desire to apply these mental tools and with them to probe and penetrate the whole mystery of existence.

And so it was that man the animal became man the philosopher, seeking with his mind and fertile, childlike imagination, the ultimate answers.

His world as yet was a simple one. It was made up of what he could see, what he could feel and experience. Surely, he reasoned, those were the things that must be basic and real, the elemental strands from which the whole fabric of being and reality were woven.

There was <u>earth</u>, basic and central. Certainly, too, there was <u>water</u>. And without question there was the magical element of fire. The Chinese pondered and added a fourth, which appeared to be different from the other three. It was metal. Still another element was named. This fifth element grew from the earth in a mysterious way: it was wood.

To put these elements together, to relate them in some kind of harmonious whole, the Chinese searched and at length settled on a powerful newfound magic object.

The story about the origin of this new magic device goes like this:

In the twenty-third century BC the phenomenal new magic pattern was revealed on the shell of a sacred turtle. This turtle crawled out of the Lo River and displayed its markings to a philosopher who by chance had stopped to rest on the river bank. Since the observation was made at the Lo River, the pattern he discovered became known as the Lo Shu.

The argument for this array of numerals having first been observed on a turtle's back is bolstered by the shape of the Chinese numerals, similar in appearance to the square incised markings on the turtle's shell, as may be observed in the illustration.

This is the simplest magic square, for the Lo Shu is an array of the integers 1 through 9 in a square subdivided into 9 squares, 3 on a side. In this square every row, every column, and each of the two diagonals total exactly the same, i. e., 15.

The two illustrations show the magic square in both Arabic and Chinese notation. Note the assignment of the five elements earlier noted, also the shading of the Yin (female) and Yang (male) principles. In this representation odd numbers are male (or heavenly); even numbers are female (or earthly). Little imagination is required to follow their reasoning in accepting the odd number configuration in terms of phallic symbolism.

Thus from early times these so-called "magic" squares were considered something more than simple mathematical exercises or arrays.

From China the concept of the array as magic spread, but before continuing the history of magic squares, let us briefly examine the Lo Shu. This is truly a gem of a mathematical device, a combinatorial curiosity of the first order.

Keeping in mind that our problem is to create an array of the integers 1 through 9 in such a way that rows, columns and diagonals all have equivalent totals, we first total 1 through 9 and find that the sum of all these numbers is 45. Therefore, since distribution must be made in three equal rows, each row must contain 1/3 of 45, or 15.

Careful examination reveals that there are only 8 separate combinations of three numbers contained in the 1 to 9 listing which yield this sum. These groupings are listed as follows:

9,5,1 *	8,4,3
9,4,2	7,6,2
8,6,1	7,5,3 *
8,5,2 *	6,5,4.*

It is immediately apparent that only the number 5 occurs in four of the possible combinations. Consequently, with the central number of necessity fitting into four separate lines, i. e., two diagonals, one column and one row, 5 obviously must serve as the center of the array.

Next, since all columns, rows and diagonals must total 15, an odd number, the positioning of odd and even numbers must guarantee odd results for all the lines. Each line, as a consequence, must contain either 3 odd numbers, or 1 odd number. The reason for this is that two odd numbers, added together, give an even number. The only configuration which gives odd totals is shown, and with 5 placed in the center, the problem is easily solved.

o	x	o
x	x	x
o	x	o

o	x	o
x	5	x
o	x	o

8	1	6
3	5	7
4	9	2

The odds were not too great against our solving this, the smallest possible magic square, by pure guesswork. The possibility is given by the 9! (9 factorial) which equals a mere 362,880 to 1.

Once started on this line of thinking the idea of magic squares spread, and squares of higher order were devised. Surprisingly, the little magic 3 square gave a recognizable pattern for solving higher order odd number squares. It contains within itself, simple as it is, two distinct methods which may be applied in solving odd squares of higher order. These are:

1. Solution through continuous northeast movement, coupled with a southern break move.

```
            8 1 6
            3 5 7
            4 9 2
. 1 . . 1 6 8 1 6
. . . 3 5 7 3 5 7
. . 2 4 . 2 4 . 2
. 1 .
. . .
. . .
```

2. Solution through a uni-directional knight's move, coupled with a southern break move. The pattern is a bit more complicated.

```
. 1 .
. . .
. . 2
      . 1 .
      3 . .
      4 . 2
      . 1 .
      3 5 .
      4 . 2
      . 1 6 . 1 6
      3 5 7 3 5 7
      4 . 2 4 . 2
            8 1 6
            3 5 7
            4 9 2
```

Now that we have derived simple algorithms to follow, let us create the next larger odd magic square, a magic-5, using the knight's move method:

.	.	1	.	.
.	.	.	.	.
.	.	.	2	.
.	.	.	.	.
.	.	.	.	3

.	.	1	.	.
4	.	.	.	.
.	.	.	2	.
.	5	.	.	.
.	6	.	.	3

.	.	1	.	.
4	.	7	.	.
.	.	.	2	.
.	5	.	8	.
.	6	.	.	3

.	.	1	.	9
4	.	7	.	.
.	.	.	2	.
.	5	.	8	.
.	6	.	.	3

.	.	1	.	9
4	.	7	.	.
10	.	.	2	.
11	5	.	8	.
.	6	.	.	3

.	12	1	.	9
4	.	7	.	.
10	.	13	2	.
11	5	.	8	.
.	6	.	14	3

.	12	1	.	9
4	.	7	.	15
10	.	13	2	16
11	5	.	8	.
.	6	.	14	3

, etc.

The result is this magic square:

23	12	1	20	9
4	18	7	21	15
10	24	13	2	16
11	5	19	8	22
17	6	25	14	3

Using the northeast method a different configuration is obtained, but with the same magic quality:

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

These are magic squares containing ordinary magic according to definition, but with something extra. Note that all the northeast diagonals of the first square total 65, while all the southeast diagonals of the second square total 65. The opposite diagonals in either case are random, with the exception of the central diagonal, which does total 65.

Now, since it would be quite satisfying to have our square contain not only ordinary magic as earlier defined but to have at the same time all diagonals both northeast and southeast to total the same 65, we find by experimentation that this indeed can be done, <u>but</u> in the odd number listing only for prime number squares (or products of primes), and these in order as high as we wish to go by changing our algorithm slightly. To accomplish this purpose we continue to use the knight's move, coupling it instead with a single space southwestern break move whenever required.
The result, for a magic 5, is:

9	12	20	23	1
18	21	4	7	15
2	10	13	16	24
11	19	22	5	8
25	3	6	14	17

Examination reveals that in this square all columns, rows and all diagonals total 65, and through use of this algorithm results remain constant for prime squares of <u>any</u> higher order which I have tried thus far, including such squares as 7, 11, 13, 19, 23, 29, 31, 37, 41, 47, and 101, as well as the products of prime numbers 25, 35, and 49. For the magic 101-square, containing the numbers 1 through 10,201, which I recently constructed in this manner, all rows, columns and diagonals have the same total, which is

515,201. Were it not for the method applying equally well to products of primes as well as primes, this method for constructing arrays could serve as a definition of a prime number. In any case it categorizes the total grouping, which one ordinarily connects by no common thread.

There has been very little in the periodicals in the last seventy years about magic squares, and practically all of it has been in Scientific American. One extremely strange trait of odd magic squares was revealed in a letter to the editor of Scientific American in 1919. The strategy, which was unnamed in the letter and was apparently the discovery of the writer, who was identified only by his initials, described a method for transforming any odd magic square into another but just as potent magic square through a process of position-number substitution. Upon experimentation, I have found that this application, which I term flexing, works perfectly on super-magic odd prime squares as well as giving through the process still another equally super-magic square.

The position-number substitution, or flexing method works in the following way:

Beginning with the super-magic-5 square developed by the knight's move coupled with a southwest break move, as pictured:

9	12	20	23	1
18	21	4	7	15
2	10	13	16	24
11	19	22	5	8
25	3	6	14	17

1. Position #1 contains 9, therefore place 1 in the 9th square,

2. Position #2 contains 12. Place 2 in the 12th position.

3. Position #3 contains 20. Place 3 in the 20th position.

5	.	.	.	.
.	.	.	1	.
.	2	.	.	.
.	.	.	.	3
.	.	4	.	.

4. Position #4 contains 23. Place 4 in the 23rd position.

5. Position #5 contains 1. Place 5 in the 1st position.

6. Continue in the same manner with the second row, etc. The completed square is as shown.

5	11	22	8	19
23	9	20	1	12
16	2	13	24	10
14	25	6	17	3
7	18	4	15	21

This method works on any prime number magic square. Flexing the resulting square produces the original square in every case.

Considering that the odds against producing any single configuration of 25 numbers by pure chance is represented by the value of factorial 25, which is: 15,511,210,000,000,000,000,000,000 (fifteen and a half septillion) to 1, this can be viewed as somewhat remarkable.

The odds against producing a magic 4 are much smaller, for 16! is only 20,922,790,000,000 (about twenty-one trillion) to 1. Still, in India these were being made, in variety, a thousand or more years ago. Again, these are things of beauty, perfect Magic 4s containing many additional features which even today are amazing.

In India, according to a progress report of the Superintendent of Hindu and Buddhist Monuments for 1915-1916 there was reported a Hindu magic square, found inscribed on a hidden portion of a lintel, brought to light by a fall of masonry, in the Chota Sarang shrine at Dudhai in the Jhansi District.

The Indian emphasis at that earlier time was on Magic-4 squares, and this square, which is reported to date from the first half of the eleventh century, is as illustrated:

7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

It is really super-magic, with all columns, all rows, and all diagonals totaling 34. What really makes this square phenomenal is that the four numbers clustered around every intersection of lines total 34 as well, and additionally there are many other surprising properties.

Still another magic-4 possessing all the above properties was discovered in India on the gate of the fort of Gwalior. It is as shown:

15	10	3	6
4	5	16	9
14	11	2	7
1	8	13	12

The Indians ascribed tremendous magical powers to these squares. Even in this century, Indian soldiers have carried metal amulets with these magic squares inscribed upon them into battle, convinced that these arrays possess magic powers to shield them from injury and death.

Because the illustrated magic 4s have all their diagonals totaling 34 they can be used to create larger supermagic squares. One example would be a magic 16 made by placing together sixteen magic 4s in ascending order. Using the Dudhai as an overall pattern, the individual blocks would be, in the 1 position, the Dudhai magic 4 itself. For the position 2 block the numbers would be 17 through 32, inserted in the same ascending sequence as the numbers in block 1, etc. It will be found that, upon completion of the large square, it will have all rows, columns and diagonals with exactly the same totals--in this case, 2,056.

An alternate approach is to establish a series, placing in block #1 in the proper configuration the series 1, 17, 33, 49 ......225, 241, created, of course, by adding 16 each time to create the series, with the first series going into block #1, then simply adding 1 to each number of the series as blocks #2, etc., are inserted. The result will be another perfect (or super-magic) 16 square.

Upon discovering and devising a number of new methods for constructing perfect, or super-magic, squares, I established a goal of creating, or at least determining a sure method for their construction) as many as possible of the perfect magic squares which can be made from 1 to 100. Notable gaps are 6, 9, 14 and 15. In the efforts so far, 59 can definitely be made. Some squares seem small and simple, but they are not. They appear not only formidable, but impossible to build. Ordinary magic for them is easy, but not super-magic, or perfection. Should you wish to try them, your help will certainly be welcomed.

It may be well here to pause and explain again that the term "magic" as used in this paper is simply a mathematical definition for particular arrays of numbers, some more "magic" than others because they possess more singular properties.

With this in mind, and with his tongue in his cheek, Benjamin Franklin, well pleased with an ingenious Magic 16 matrix of his own creation which will be discussed presently, praised it, declaring that it was "the most magically magical of any magic square ever made by any magician."

This towering figure in our American history enjoyed magic squares and developed great proficiency in creating them. Benjamin A. Franklin, patriot, printer, statesman, inventor, electrician, and renowned ladies man, derived great pleasure from creating unique squares and any brief report on such squares is compelled to mention him.

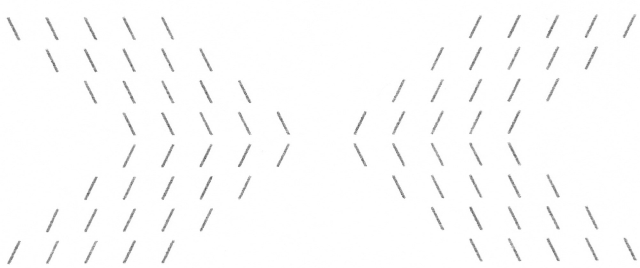
An account of Franklin's squares is given in Carl Van Doren's excellent biography. Two representative squares are reproduced.

Franklin possessed an inquiring, fertile mind. It is unfortunate that as schoolchildren we learn only of his kite and key experiment, and do not learn that he alone is responsible for much of the vocabulary which applies to electrical matters today. Van Doren presents an astounding list as he tells us "Franklin appears to have been the first to use, at least in print in English, these electrical terms: armature, battery, brush, charged, charging, condense, conductor, discharge, electrical fire, electrical shock, electrician, electrified, electrify, electrilized, Leyden bottle, minus (negative or negatively), negatively, non-conducting, non-conductor, non-electric, plus (positive or positively), stroke (electric shock), uncharged. The Philadelphia Prometheus with his kite was also an American Adam in his electrical garden."

Returning to the squares, there are two illustrations, one a Magic 8 and the other a Magic 16, but these squares are different from the old run-of-the-mill squares. Apparently Franklin had grown tired of the same old magic squares which were, for him, so easy to make that he could fill the spaces as fast as he could write the numbers. His unique matrices give us a window into the complex mind of Franklin, who found great delight in experimentation and invention, and in toying with intricacies.

Franklin's Magic-8

52	61	4	13	20	29	36	45
14	3	62	51	46	35	30	19
53	60	5	12	21	28	37	44
11	6	59	54	43	38	27	22
55	58	7	10	23	26	39	42
9	8	57	56	41	40	25	24
50	63	2	15	18	31	34	47
16	1	64	49	48	33	32	17



Characteristics: Bent diagonals, as indicated in the chevron shapes, all total 260, the same as rows and columns. The effect can be extended by scrolling. Any 4 grouped numbers total 130. Half rows as well as half columns total 130.

The same method of construction is employed in his Magic 16, and in the 16 square the same chevrons pointed in all directions give the correct total of 2,056, while one may cut out a paper frame of a proper size to expose a square of 16 numbers at any random position upon the Magic 16, and observe that in every case the sum of exposed numbers is the same 2,056.

This is his remarkable Magic-16 Square.

(illustration)

In the search for perfection or supermagic I used Franklin's 8 and 16 squares, for it is obvious that by reversing one side and reordering from bottom to top, perfect magic 8s and 16s can be created, as shown. Far easier to make are the Indian even squares, however, and they possess the advantage of working as well for the intermediate numbers 12, 20, etc., which cannot be formed by the Franklin method. The 12, made by an extended application of the Gwalior magic 4 pattern, is shown below:

(illustration attached)

More information is contained in the sheets which have been given you, as we have barely touched on the subject. For those of you who enjoy cryptograms, crossword puzzles, and baffling mathematical problems it is suggested that a brief examination of magic squares as a possible source of mental challenge and pleasure for you may prove worthwhile.

As a defense of the topic, upon one occasion termed by another as "trifling and useless," let Benjamin Franklin be my spokesman. His comment was: "Perhaps the considering and answering such questions may not be altogether useless, if it produces by practice an habitual readiness and exactness in mathematical disquisitions, which readiness may on many occasions be of real use."

Graph paper has been provided each of you together with the supplemental information, in case you are venturesome and find that you are possessed by an overwhelming desire to start at once and create your own magic arrays. Pencils are suggested, preferably pencils equipped with large erasers. It is hoped that you will discover hidden talent and prove yourselves to be able magicians. Good luck!