Nov. 1957?

BEYOND THE THIRD "R"

Tom Soyar

Sputnik with its eerie "beep - beep - beep" and its "Made in Russia" label symbolizes and dramatizes the fact (and and I feel sure "fact" is the correct word) that in many respects American education is inferior to the Russian brand. One of the gravest deficiencies in our educational system is that the average American student gets far less science and mathematics than does his Communist counterpart. Many reasons can be advanced as to why the modern American student shuns math courses as he would the plague. In my opinion one of the principal ones is that he has grown up in a society where mathematics is generally looked down on, regarded as a dry, boring, tedious, uninteresting, extremely difficult subject studied only by bearded, anemic, bespectacled creeps and "mad scientist" types from a B-grade science-fiction movie. It is the purpose of this paper to show that this widely held belief of the American public is erroneous; that higher mathematics is, or ought to be, if properly taught, a bright, entertaining, interesting and at times fascinating subject. I shall attempt to do this by relating (in what I fear is a rather disjointed fashion) a few of the facts and theories of higher mathematics which I believe will support my thesis. Before doing this I think it only fair to warn you (probably to your great relief) that I do not have a technical knowledge of mathematics, that I have never taken a math course beyond the high school level and that some of the ideas I'm advancing tonight I have accepted purely on faith.

Perhaps as good a starting place as any is a discussion of large numbers. Many interesting stories have been told discussion of large numbers. Many interesting stories have been told illustrating just how immense certain numbers are. One of these deals with the ancient Indian sultan who desired to reward one of his subjects for inventing the game of chess. The king offered to give the inventor anything he desired. The man's request was

a seemingly modest one. He wanted to be rewarded in wheat, and he wanted a grain of wheat for the first square of a chess (or checker) board, two for the second, four for the third, eight for the fourth, and so on doubling the number of grains on up through the sixty-fourth square. Granting the request seemed simple enough until the sultan started counting. It developed that it would require over eighteen quintillion (18 followed by eighteen zeroes) grains to fulfill the promise. This number is estimated by one authority (Meyer, "Fun With Mathematics") as being sixteen thousand times the average annual U. S. wheat crop! The story as it is usually told ends here but one is tempted to believe that instead of getting his wheat the inventive gamester got the ax for using his mathematical knowledge to confound his august majesty.

Another very large number is described in the famous "Problem of a Printed Line". For this project we are going to try to calculate how many different things can be written on a standard 65 space line of printing. Since in English there are 50 different commonly used symbols (including the 26 letters of the alphabet, the 10 figures and 14 punctuation marks), this will require us to arrange these 50 different characters in all possible combinations in the 65 space line. This number can all possible combinations in the 65 space line. This number can be expressed as 50^{65} (i.e. 50 multiplied by itself 65 times) and works out to be approximately 1 followed by 110 zeroes. A more graphic way of understanding the size of this number is to view it in this way. Assume that every atom in the universe is a printing press; that all are working simultaneously and have been since the universe was created (some three billion years ago) and that they print at the rate of atomic vibrations (i.e. 15 quadrillion lines a second). If so -- we would be approximately 1/30 of 1% through with our job.

-2-

You will recall that above we mentioned the number of atoms in the universe. Yes, mathematicians have estimated their number (3×10^{74}). In fact Eddington, the British physicist, has estimated the number of <u>electrons</u> in the universe (157×10^{77} or 157 followed by 77 zeroes.)

One of the largest numbers which has a name is the "googol". This number is 1 followed by 100 zeroes. The number got its name in an interesting way. Dr. Edward Kasner of Columbia University was trying to coin a name for the number under discussion. While Dr. Kasner was puzzling over the problem his six year old nephew wandered into his study and in a fit of desperation Dr. Kasner asked his nephew what he would call the figure 1 followed by 100 zeroes. Without batting an eyelash the boy answered "a googol". The name stuck. Later Dr. Kasner asked the youngster why he hit upon that name. The boy with the withering logic of the six year old, replied, "It just looked like a googol" (Newman, "The World of Mathematics, Vol. 3 page 1007). Professor Kasner also created the even larger number, the googolplex. This is the figure 1 followed by a googol zeroes. The words "googol" and "googolplex", while not in the vocabulary of the average person by any means, are nevertheless good English words and not purely the property of the higher mathematicians. Both will be found in any reasonably comprehensive desk dictionary.

A number so amazingly large that it has almost no meaning can be easily written with 3 figures, thus --9^{9^9}. This is the equivalent of 9 multiplied by itself over 387 million times. It would contain several hundred million digits; printed in ordinary type it would stretch for thousands of miles. Nobody, even today with the aid of electronic calculators, has ever computed exactly what this number is, but its first nine digits are 428, 124, 773 and its last two are 89. The difficulty of determining the intermediate digits

of this vast number is so great that mathematicians are content to follow the advice of the popular song of a decade ago, "Don't Mess with Mr. In Between".

The numbers we have been discussing are very large, but are still less than the numbers we are now going to examine. For all of the numbers mentioned above, while inconceivably immense in terms of our everyday world, are still finite. We now turn to the ultimate of numbers -- infinity. We are all aware that there are certain quanities which are infinite. The series of numbers is infinite, since no matter how large a number one is speaking of, there are still others larger. A first grade child with a single stroke of his little pencil could write a number larger than that written by the most brilliant mathematician of all time provided the mathematician were compelled to "write his number first". If you remember even the rudiments of your high school geometry you will recall that there are an infinite number of points in a line and an infinite number of lines in a plane. These concepts, while complex in some ways, seem relatively simple when compared with some of the more abstract notions of higher mathematics. And so they seemed until George Cantor, the 19th Century French mathematician, set his ingenious mind to work on the concept of infinity. The question Professor Cantor kept asking himself seems patently ridiculous but Cantor asked it anyway, "Are some infinities larger than others?" It is easy to see the difficulties Cantor faced in trying to show that there were some infinities larger (or as it is sometime said stronger) than others. For how can we tell which of two numbers is larger when both are so large that we cannot count either of them? Professor Cantor avoided this difficulty by showing that it is not absolutely necessary to be able to count two quantities to determine which if either of them is larger. He showed that this was possible by the following analogy.

Cultural anthropologists long ago discovered that there are certain primitive tribes, for example the Hottentots, who cannot count beyond three. If you asked a Hottentot how many enemies he had slain or how many sheep he had and the number was greater than three he would reply "Many". Regardless of whether the actual number was four, five, ten or a hundred, his answer would be the same, "Many." But because the Hottentot cannot count beyond three, does that mean that he could not tell, for example, that 9 was larger than 8? It does not. For the Hottentot, if he is clever enough, can determine that 9 is greater than 8 by the simple process of pairing off. For example, if he wants to determine whether he has more beads (of which he actually has 9) than bracelets (of which he has 8) he simply puts one bead and one bracelet side by side and continues doing so until he discovers thathe has a bead left with no bracelet to pair off against it. He therefore correctly concludes that he has more beads than bracelets, even though the absolute numbers involved in the problem are beyond his counting comprehension.

It was with this same method that Cantor began to compare the sizes of infinities. His first experiments merely served to provide further evidence for the generally accepted notion that all infinities were equal. For example, Cantor showed that the number of numbers was equal to the number of odd numbers (or of even number of numbers was equal to the number of odd numbers (or of even numbers); that the number of points in a short line was the same as the number in a long line; that the number of points on a line was the same as the number of points on a plane. But Cantor was finally able to detect some differences. He was able to show (to his own satisfaction and to the satisfaction of most other mathematicians, though not necessarily to the writer of this paper) that the number of points on a line was greater than the number of numbers. His proof is too intricate to go into here in detail ~~but~~ but briefly

stated it was this. He was able to show that the number of whole numbers was the same as the number of fractions. Since all fractions when converted to decimals become repeating decimals (i.e., they begin to repeat a single digit or series of digits, e.g. 1/3 equals .33333 etc.) it therefore followed that the number of repeating decimals was the same as the number of numbers, and since a point on a line can be expressed geometrically as a <u>non</u>-repeating decimal, it then follows that there are more points on a line than there are numbers! Our two infinities are not equal!.

By a still more involved bit of mathematical reasoning, Cantor showed that the number of curves, including of course those of the most peculiar shape imaginable, was even greater than the number of points on a line. Beyond this Cantor could not go and even today the largest number known to mathematics is the number of possible curves.

Having reached the ultimate point in our discussion of large numbers, we now proceed to that branch of mathematics known as statistics and probability. Nearly all of us have some idea of simple probabilities. For example, we all know that there is one chance out of two that a tossed coin will fall "heads," or to phrase it in gambling lingo, the odds are one-to-one that it will be heads. Those of us who are card players or crap shooters will, if we are even slightly skilled, know a great deal more probability theory than that.

In fact, the entire branch of mathematics known as probability (and therefore many other discoveries in both the natural and the social sciences) owe a great debt to those old vices - card playing and crap shooting. During the latter years of the Middle Ages, playing cards were introduced to the royal courts of Europe, - apparently, though this is uncertain, from wandering gypsies. Card games naturally created a sort of vague awareness of some of the general principles of probabilities. Finally in the 17th century an absolutely rigorous theory of probability was developed when a crap shooting friend urged the French mathematician-philosopher, Blaise Pascal, to figure out the odds on throwing a particular number. Pascal did so, and his imagination stimulated, went on to lay the foundation for the science known as probabilities.

Another factor which is often cited as contributing to the development of the science of probabilities is the growth of insurance as a commercial institution. In order to know what premium to charge a person insuring against certain calamities one would have to know how probable it would be that they would occur. The influence of insurance on the development of probabilities is strongly emphasized by historians with a Marxist bent who like to attribute all discoveries to profound changes in economic institutions and who would never admit that an entire branch of mathematics owes its existence to such frivolous pastimes as playing cards or shooting dice.

Some of the truths of probability mathematics are so amazing as to almost be unbelievable, so unbelievable in fact that a good mathematician, if he is a betting man, can pick up a little change on the side from his gullible friends. For example, suppose that two people each have a complete deck of cards. They deal these cards out one at a time, first player one, then player two. What do you suppose would be the probability that some time during the course of the dealing they would both turn over the same card at the

... dealing they would both turn over the same card at the same time? A great many people would consider this extremely unlikely but in fact the odds are almost two to one that such an event will happen, or in other words, that it will happen on the average two out of three times.

Another example - What do you think would be the odds against making five "pat" poker hands from any twenty-five

cards chosen at random from the deck; five hands of nothing but straights, flushes and full houses? Many people will lay long odds that no such arrangement can be made, but actually the feat can be performed almost every time as was demonstrated by the T-V cowboy star Maverick on his show a couple of weeks ago when he used it to win a bet. Testing it myself I hit 6 straight times.

An even easier way of winning a bet is by making the wager of the coinciding birthdays. For this wager we need at least twenty-four people present. We then pose the following question. Suppose I agree to bet that, out of the twenty-four people here, at least two of you have the same birthday, e.g. there will be two people born on Sept. 7 or May 8 or June 10, etc. What odds will you give me on this? The writer has posed this question several times to his friends and they have offered odds ranging from 5 to 1 to 30 to 1. Actually, however, the odds are in favor of there being at least 2 people out of 24 who have the same birthday. To be exact the odds are 54 out of 100.

This statement seems so unbelievable that I felt I should test it before presenting it here tonight. For the sake of simplicity, I did not actually call together 24 friends and ask them their birthdays. Instead I merely ran down the names in a biographical dictionary, "Meacham's History of Christian County".

I began with Henry Abernathy, the first name. The first twenty four birthdays listed did not show any two alike. In fact it was necessary to go to the 29th name, Austin Bell, to get the first pair. He and C. R. Atkins were both born on April 16. But in the second 24 names checked (from T. J. Baynam to Alvin Clark) I found three pairs of birthdays; William Campbell, Erin Campbell and Eliza Campbell [redacted] were all born on December 12 and J. P. Campbell, II and J. P. Campbell V were both born on December 8. I checked the birthdays of the Presidents. Sure enough, two of them, James K. Polk and Warren G.

Harding had the same birthday, November 2. Since death days will do as well as birthdays when one uses a biographical dictionary, I checked the death dates of our thirty deceased presidents. Filmore and Taft died on March 8th. Jefferson, John Adams and Monroe all died on July 4th, Jefferson and Adams in fact on the same day, July 4, 1826. If you can make the wager of the coinciding birthdays often enough you will die a rich man. You won't win every time but you will have odds going in your favor far stronger than those that provide a lush living for the gambling casino operators of Monte Carlo and Las Vegas.

A type of number that has caused a great deal of mathematical speculation is the prime number. A prime number is a number which cannot be divided by any other number, except, of course, itself and one. For example, one, two, three, five, seven, eleven, thirteen and seventeen are [redacted] primes. Although Euclid in the third century B. C. showed that there was no such thing as <u>the</u> largest possible prime, mathematicians have spent much time and energy looking for larger and larger primes. In the late 19th century the French mathematician, Edouard Lucas, discovered a 39 digit prime. But in 1952 a giant electronic calculator discovered a 687 digit number which was a prime . ($2^{2281} - 1$ or 4.461×10^{686}) As far as I know this is the largest known prime.

Formulas have been sought for producing primes. The Frenchman, Fermat, discovered one in 1640 which for many years was thought to produce only primes. But a hundred years later the German, Euler, showed that the number 4,294,967, 297 which acording to Fermat's table should have been a prime was not, but was, in fact the product of 6,700,417 and 641. How he discovered this, I don't know, but if he did by trial and error I expect the project kept him pretty busy.

An interesting problem involving primes is the

called Goldbach conjecture, proposed in 1742, which states that any even number can be expressed as the sum of two primes. By examining the lower even numbers we can see that this is true. 4, an even number, is the sum of 3 and 1, both primes. 6 equals 5 plus 1; 8 equals 5 plus 3 or 7 plus 1, etc. No even number has been found that violates the rule proposed by the Goldbach conjecture. But no rigorous proof of the rule has been discovered. The Goldbach conjecture states that all even numbers can be expressed as the sum of <u>two</u> primes. The Russian, Schnirelman proved that every even number was the sum of not more than 300,000 primes. Another Russian, Vinogradoff, lowered this number to four, but in spite of the fact that four is a considerable improvement over 300,000 we are still not at the <u>two</u> which the Goldbach conjecture and common experience say is enough.

Another branch of higher mathematics which deals with some interesting matters is Topology, defined (very generally) as the study of surfaces. The entire field of topology dates back to the 19th century when the German mathematician, Mobius, noticed that a certain type of paper ring (now called a Mobius strip) exhibited certain remarkable qualities. A Mobius strip can be simply made by taking a long strip of paper and gluing it into a ring making a half twist in the paper before joining the ends. One of the properties of this figure is that it has only one side and one edge. This seems impossible since we should certainly think that an ordinary paper ring would have two sides and two edges. But one can verify that the Mobius strip actually has only one side, (and one edge) by running his finger around it. Sure enough, you can go all around the entire strip and return to your starting place without ever lifting your finger. Another interesting property of the Mobius strip is that if it is cut down the middle it will not be cut into two rings as you might expect, but in fact will become one long narrow ring. And if this ring is similarly divided down the middle you will then

get your two rings but they will be interlocked like two links in a chain.

Another interesting property of the Mobius strip is that when an object is passed around its surface its profile is reversed. That is, if the object started out around the Mobius strip facing in one direction it would return facing in the opposite direction. This is difficult to explain without detailed illustrations and I shall not attempt it here. However, an interesting ramification of this phenonomen may appeal to you. If our three dimensional universe is curved in a Mobius way, as some mathematicians say that it is, a person circumnavigating the universe would come back with his heart on the right side of his body, left handed instead of right handed and in general turned around. This sounds incredible but it is exactly what happens when a glove is turned inside out. A right handed glove becomes left handed and vice versa. Professor George Gamow of George Washin that the glove and shoe manufacturers of the future will have to produce only right handed gloves and right footed shoes. They can then send half of the gloves and shoes on a rocket ride around the universe and get them back as left handed gloves and left footed shoes. However, Gamow concludes tongue in cheek that even if theoretically possible such a method of manufacturing shoes and gloves would not be very practical

Another interesting problem of topology is the so called "problem of the four colors". Imagine a map, and then try to figure how many colors will be needed to color it so that no two adjacent regions will have the same color. By a few minutes experiment you will find that any map you can draw can be properly colored with only four colors. No one has ever succeeded in drawing a map that cannot be colored with four colors and it is safe to say that no map will ever be drawn that violates the rule of the four

colors. But no mathematical proof of this has ever been devised. The problem of the four colors has a special appeal for me because back in grade school days when bored with ~~the~~ my lessons I would spend hours trying to draw a map which couldn't be colored with four colors in order to obtain the $100 reward my father had promised me if I succeeded.

All of us realize that there is more than one number system. For example, we are all familiar with Roman numbers. As we all know our own Arabic system is a much superior system. For example, multiplication and division are far easier in the Arabic system than in the Roman. We are prone to look on our Arabic system which is, of course, a decimal system as just about perfect. However, for certain purposes it is quite inadequate and other systems have been devised. For example, let's briefly examine a system in which it was necessary to devise in order to [redacted] make electronic calculators practical. This is the binary system, and it is based on the number two instead of the number ten. It has only two digits, zero (0) and one (1). Time does not permit a full discussion of this system but as you can see from this chart, every number is written by using only the two figures 0 and 1. The reason why this system is necessary for calculating machines is that calculators use punch cards and electromagnetic relays in much of their work. Since there are only two digits in the binary system, we can "punch out" any number on our punch card by letting a hole represent 0 and "no hole" represent 1, or vice versa. In the intricate system of electronic relays, having the relay on can represent 0; off, 1 or the other way around. To be sure, it is a little longer and a little harder to write out the numbers and set up the problems in the binary system than in the decimal system but the immense rapidity with which electronic calculators solve the problems once set up makes up for this difficulty a hundred times over.

In this paper I have tried to show some of the interesting facts, theories and speculations that exist in the field of mathematics. It is my belief that if some of this information could be imparted to our American youngsters a lot more of them might be interested in going into mathematics and its related fields. This is not a plea for more "sugar-coated" learning. But it is a plea for a new approach to teaching mathematics. It is my own belief that too many of today's teachers hide the beauty, the symmetry, the harmony, yes, even the drama of mathematics behind an all but inpenetrable fog of X's and Y's and apples and oranges. I believe we all agree that some change should come. Let's hope it's not too late.

B I B L I O G R A P H Y

Gamow, George, "One, Two, Three, -- Infinity," Mentor Books, New York, N. Y. 1947.

Meyer, Jerome S., "Fun With Mathematics." Fawcett Publications, Greenwich, Conn. 1952.

Newman, James R. "The World Of Mathematics," 4 Vols. Simon & Shuster, New York, N. Y. 1936.

Hogben, Lancelot, "Mathematics For The Millions," W. W. Norton & Co., New York, N. Y. 1937.

Assorted Issues of "Scientific American," May 1957 to Jan. 1958.